Quantification of Reactor Kinetics Parameters during Reactor Transients using Cherenkov Light and Auxiliary Application to Nuclear Safeguards

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Dissertation Research Proposal

Assemble off-the-shelf instrumentation in a specific, novel configuration to measure Cherenkov radiation during normal reactor operations and interpret data with fundamental reactor kinetics relationships to determine occurrence of plutonium diversion.
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Motivation

• IAEA and NNSA have an interest in the fissile material content in all reactors worldwide
  o Monitoring for Nonproliferation Treaty (NPT) – 1968
  o Additional Protocol – INFCIRC/540 – 1997

• IAEA is tasked with verifying that signatories of NPT comply with their commitments
  o State could be the adversary

• Ideal monitoring systems are non-intrusive
  o Nondestructive Assay (NDA)

• IAEA is currently limited in its inspections of research reactors
NDA Characteristics

- Phenomenon during normal operations is measured

Cacophony of neutral and charged particles with varying energies is produced
Cherenkov Radiation

With attenuation in water, UV is preferentially absorbed.

adapted from Ilver (1993)
Cherenkov Radiation

With attenuation in water, UV is preferentially absorbed
Previous Work – Tehran Research Reactor

- Previous work has shown that Cherenkov light is proportional to reactor power

- Experiments performed at Tehran Research Reactor (TRR) in 2009
  - Measurement was sensitive to changes in environment
  - Attempted to directly correlate photon intensity to Watts

- Could not resolve issues
Proposed Research – Previous Work

• TRR is not capable of pulsing
  o Observed Cherenkov radiation is proportional to power

• In a reactor capable of pulsing, the ratio of Cherenkov radiation intensities between two reactivity states will provide ratio of reactor powers

• Research reactors of interest must:
  o Be capable of performing reactor transient (i.e. pulsing)
  o Allow instrumentation to have visibility of the core (open-pool)
Dissertation Research Proposal

Assemble off-the-shelf instrumentation in a specific, novel configuration to measure Cherenkov radiation during normal reactor operations and interpret data with fundamental reactor kinetics relationships to determine occurrence of plutonium diversion.
Reactor Experiments

Cherenkov radiation
Reactor Experiments

Off-the-shelf instrumentation
Reactor Experiments

![Graph showing PD Response (mV) vs. Time (ms)]

- Wavelength (nm)
- Photon Intensity
- Cherenkov Photons Produced
- Water Surface
- 1.88 m Below
- 3.21 m Below
- 4.50 m Below

![Image of reactor experiments equipment]

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Reactor Experiments

Kinetics Relationships?
Point Reactor Kinetics

• A simple, fundamental reactor kinetics relationship is desired to determine plutonium diversion

• If delayed neutrons are ignored and temperature feedback parameters are assumed constant, the Fuchs-Nordheim (FN) model is found:

\[
\frac{dP}{dt} = \frac{\rho - \beta}{l} P(t)
\]

\[
\rho = \rho_0 - \alpha T
\]

\[
\frac{dT}{dt} = KP
\]

\[
P(t) = P_{\text{max}} \text{sech}^2 \left( \frac{\rho_0 - \beta}{2l} t \right)
\]

\(P\) – Reactor power
\(\rho\) – Reactivity insertion
\(l\) – Prompt neutron lifetime
\(\beta\) – Delayed neutron fraction
\(T\) – Temperature
\(t\) – Time

\(\alpha, K = f (T)\)

Fundamental reactor kinetics relationship
Point Reactor Kinetics

• The point reactor kinetics equation is a function of:
  o Reactivity inserted for perturbation ($\rho$)
  o Delayed neutron fraction ($\beta$)
  o Prompt neutron lifetime ($l$)

• Response of reactor to a reactivity perturbation is dependent on these factors

• Delayed Neutron Fraction
  o $\beta = 0.00686$ for U-235
  o $\beta = 0.00224$ for Pu-239

• Observe difference between U-235 and Pu-239
Reactor Experiments – Results

Glasstone (1970)

Hetrick (1993)
Reactor Experiments – Results

To compare pulses, align the peaks!
\[
\frac{P(t)}{P_{\text{max}}} = \text{sech}^2 \left( \frac{\beta (\$ - 1)t}{2l} \right)
\]

\[\$
= \frac{\rho}{\beta}\]
Reactor Experiments – Results

\[ \frac{P(t)}{P_{\text{max}}} = \text{sech}^2 \left( \frac{\beta (S - 1) t}{2l} \right) \]

Major outcome

\[ \frac{P(t)}{P_{\text{max}}} = \text{sech}^2 \left( \frac{\beta (S - 1) t}{2l} \right) \left( 1 + H(S - 1)^2 \right) \]
Reactor Experiments – Results

$2.00 Pulses Conducted at OSTR
Reactor Experiments – Results

$2.00$ Pulses Conducted at OSTR
Reactor Experiments – Results

$2.00 Pulses Conducted at OSTR

![Graph showing normalized power over time with different lines representing various conditions and parameters.]

- FN BOL
- FN + 2σ
- FN - 2σ
- P-18
- P-20
- P-23
- P-27
- P-30
- OSTR IC
Reactor Experiments – Results

• Determining $\beta/l$ ratio
  • Delayed neutron fraction ($\beta$)
  • Prompt neutron lifetime ($l$)

• Ratio provides
  • Fissile material content
    • Delayed neutron fraction different for U-235 and Pu-239
  • Reactor geometry

\[
\frac{P(t)}{P_{\text{max}}} = \sec h^2 \left( \frac{\beta (S-1)t}{2l} \right) \left( 1 + H(S-1)^2 \right)
\]
Reactor Experiments – Results

• Integrate from peak to end of pulse
  • Only dependent on reactor kinetics parameters

• Two different $\beta$ reactor pulses
  • Assume $H$ is constant

\[
I = \int_0^\infty \frac{P(t)}{P_{\text{max}}} \, dt = \int_0^\infty \text{sech}^2 \left( \frac{\beta (\beta-1) t}{2l} \right) \left( 1 + H (\beta-1)^2 \right) \, dt
\]

\[
I = \frac{2l}{\beta (\beta-1) \left( 1 + H (\beta-1)^2 \right)}
\]

\[
\frac{\beta}{l} = \left( \frac{1}{(\beta_2 - 1)^2 - (\beta_1 - 1)^2} \right) \left( \frac{2(\beta_2 - 1)^2}{I_1 (\beta_1 - 1)} - \frac{2(\beta_1 - 1)^2}{I_2 (\beta_2 - 1)} \right)
\]
Reactor Experiments – Results

- Integrate from peak to end of pulse
  - Only dependent on reactor kinetics parameters

- Two different $ reactor pulses
  - Assume $H$ is constant

\[ I = \int_0^\infty \frac{P(t)}{P_{\text{max}}} \, dt = \int_0^\infty \text{sech}^2 \left( \frac{\beta ($ - 1) t}{2l} \right) (1 + H ($ - 1)^2) \, dt \]

\[ I = \frac{2l}{\beta ($ - 1)(1 + H ($ - 1)^2)} \]

\[ \frac{\beta}{l} = \left( \frac{1}{(l_2 - 1)^2 - (l_1 - 1)^2} \right) \left( \frac{2(l_2 - 1)^2}{I_1 (l_1 - 1)} - \frac{2(l_1 - 1)^2}{I_2 (l_2 - 1)} \right) \]
Reactor Experiments – Results

<table>
<thead>
<tr>
<th>Pulses Compared</th>
<th>$\beta/l$ ratio</th>
<th>Uncertainty (2σ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>OSTR</td>
<td>336.28</td>
<td>43.38</td>
</tr>
<tr>
<td>P18 – P29</td>
<td>336.90</td>
<td>6.41</td>
</tr>
<tr>
<td>P22 – P29</td>
<td>332.04</td>
<td>7.28</td>
</tr>
<tr>
<td>P23 – P29</td>
<td>334.25</td>
<td>6.51</td>
</tr>
<tr>
<td>P30 – P29</td>
<td>336.96</td>
<td>6.40</td>
</tr>
</tbody>
</table>
Proposal – Proliferation Resistance

Assemble off-the-shelf instrumentation in a specific, novel configuration to measure Cherenkov radiation during normal reactor operations and interpret data with fundamental reactor kinetics relationships to determine occurrence of plutonium diversion.
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\[ \beta \]

\[ l \]
Proliferation Resistance

In a material diversion scenario, an operator could adjust reactivity insertion to accommodate differences in fissile material content ($-1) \rightarrow \text{N}(-1)$

\[
\frac{I_1}{I_2} = \frac{\int_0^\infty \frac{P_1(t)}{P_{\text{max},1}}}{\int_0^\infty \frac{P_2(t)}{P_{\text{max},2}}} = \frac{N_2 \left( \frac{s_2}{s_1} - 1 \right) \left( 1 + HN_2^2 \left( \frac{s_2}{s_1} - 1 \right)^2 \right)}{N_1 \left( \frac{s_1}{s_1} - 1 \right) \left( 1 + HN_1^2 \left( \frac{s_1}{s_1} - 1 \right)^2 \right)}
\]

\[
\frac{P_{\text{max},1}}{P_{\text{max},2}} = \frac{N_1^2 \left( \frac{s_1}{s_1} - 1 \right)^2 \left( 1 + HN_1^2 \left( \frac{s_1}{s_1} - 1 \right)^2 \right)}{N_2^2 \left( \frac{s_2}{s_2} - 1 \right)^2 \left( 1 + HN_2^2 \left( \frac{s_2}{s_2} - 1 \right)^2 \right)}
\]

<table>
<thead>
<tr>
<th>Dimension</th>
<th>Quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Detectable Fraction of $\beta$/$\lambda$ Ratio</td>
<td>0.986</td>
</tr>
<tr>
<td>Detectable Fraction of Fissile Material (g)</td>
<td>198.4</td>
</tr>
</tbody>
</table>
Reactor Experiments – Conclusions

• Novel, simple analysis technique for reactor pulses
  • Methodology is most impactful outcome
  • Quantifiable uncertainty
  • Shown to be repeatable
  • Cherenkov light has been proven to provide accurate representation of reactor power during transients

• Additional impact as safeguards technique
  • Method provides a new tool for safeguards inspections in support of global nonproliferation
Thank you for your attention.
Reactor Experiments – Results

\[ \frac{P(t)}{P_{\text{max}}} = \text{sech}^2 \left( \frac{\beta (\$-1) t}{2l} \right) \]

\[ \frac{dP}{dt} \bigg|_{t=0} = \frac{\beta (\$-1)}{l} \]

Normalized Reactor Power

Time (milliseconds)
Reactor Experiments – Results

\[ \frac{P(t)}{P_{\text{max}}} = \text{sech}^2 \left( \frac{\beta (\$ - 1)t}{2l} \right) \]

\[ \frac{dP}{dt} \bigg|_{t=0} = \frac{\beta (\$ - 1)}{l} \]
Reactors Experiments – Results

\[ \frac{P(t)}{P_{\text{max}}} = \text{sech}^2 \left( \frac{\beta (\$ - 1) t}{2l} \right) \]

- Scalettar (1963)
  - Varying heat capacity
  - Constant reactivity feedback
  - Deviation from FN
  - Difficult to quantify

\[ \frac{P(t)}{P_{\text{max}}} = \text{sech}^2 \left( \frac{\beta (\$ - 1)t}{2l} \left( 1 + H (\$ - 1) \right) \right) \]
Proposed Research – Requirements

• Research reactor required, and needs to be perturbed from steady-state
  o Normal operations for research reactors
    ▪ Oregon State TRIGA Reactor (OSTR)
    ▪ Square wave or reactor pulse

• Measure Cherenkov radiation intensity as a function of time following perturbation

• Obtain information about reactor fuel
  o Fissile material content
Proposed Research - Requirements

- Cerenkov Radiation Assay of Nuclear Kinetics (CRANK) system

- Instrumentation should be:
  - Light
  - Inexpensive
  - Ready-to-order

- Additionally, system should not require the use of a library of research reactor kinetics parameters

- Measurements should be:
  - Simple to perform
  - Easy to interpret
  - Understood by inspectors who are not experts in reactor kinetics
Turning the “CRANK” system

- Instruments will be required to:
  - Respond rapidly (~ ms)
  - Be sensitive to wavelengths characteristic of Cherenkov radiation
Cherenkov Radiation

- In water, photons caused by de-excitation are predominantly in the UV spectrum, but humans only see blue-violet.
Prompt Jump Approximation

- Prompt jump (PJ) approximation reduces the point reactor kinetics equations to an analytical solution

- Easily Solvable
  - Does not include temperature feedback terms
  - Solution is divided into two exponential terms

- During reactivity insertion, the normalized response of reactor, \( P(t) \), is characteristic of fissile material contained in reactor

\[
P(t) = \frac{\beta}{\beta - \rho} e^{\frac{\lambda \rho}{\beta - \rho} t} - \frac{\rho}{\beta - \rho} e^{\frac{\rho - \beta}{\Lambda} t}
\]
Prompt Jump Approximation - Kinetics

- If “compensated” reactivity is inserted...
  - Can still detect falsified declarations

- Only exponential of second term is different!
  - Decays away quickly (~ms)
  - More obvious with larger reactivity insertion

- Resolving this term requires fast signal collection

\[
P(t) = \frac{\beta}{\beta - \rho} e^{\frac{\lambda \rho}{\beta - \rho} t} - \frac{\rho}{\beta - \rho} e^{\frac{\rho - \beta}{\Lambda} t}
\]

This term is difficult to falsify
Analytical Results – PJ Approximation

With Compensated Reactivity Insertion ($0.50)$

\[ P(t) = \frac{\beta}{\beta - \rho} e^{\frac{\lambda \rho - t}{\beta - \rho}} - \frac{\rho}{\beta - \rho} e^{\frac{\rho - \beta}{\Lambda} t} \]
Reactor Experiments – Results

![Graph showing Photodiode Response (mV) over Time (seconds)]
Reactor Experiments – Results

$0.80
Charged Particles in Water

• If a charged particle’s velocity exceeds the speed of light in the surrounding medium,
  - Atoms nearby are polarized
  - Atoms then return to de-excited state, energy is given off in the form of photons

For electron traveling through water, threshold is 262 keV
• The light produced as a result is deemed “Cherenkov radiation”
• Main contributors are Compton electrons
**Instrumentation**

- PDA25K Gallium Phosphide Photodiode
- Sensitive for 150 – 550 nm
- 1 ns – 100 μs rise time
Digital Cherenkov Viewing Device

• In fact, the IAEA utilizes Cherenkov radiation for NDA of power reactor fuel
  o Digital Cherenkov Viewing Device (DCVD)
  o Measures Cherenkov radiation for spent fuel verification

• Once fuel reaches end of life, it is placed in spent fuel pool
  o DCVD examines fuel for missing fuel rods
  o Verify operator declarations
    ▪ Burnup
    ▪ Cooling time

DCVD, per Channel Systems
DCVD Disadvantages

• However, several drawbacks to the DCVD
  o Heavy!
  o Expensive (single unit ~$250k)
  o Requires library of previously known spent fuel assemblies with varying burnups and cooling times
  o Not appropriate for operating reactor
  o Lengthy time between order and receipt of instrument

• Cannot readily be utilized in measurements for proposed research

• Proposed research will expand IAEA’s capabilities, not replace DCVD
Reactor Experiments – Results

Monday

Wednesday
Reactor Experiments – Results
Reactor Experiments – Results

The graph shows the variation of Core Excess ($) on different days of OSTR Operations. The x-axis represents the day of the week (Mon, Tues, Wed, Thurs, Fri) and the y-axis represents the Core Excess ($) from 2.25 to 2.7. The data points are scattered across the graph, indicating fluctuations in Core Excess on each day.
Reactor Experiments – Results

High Core Excess – Monday
Low Core Excess – Wednesday
IAEA & Research Reactors

• IAEA is currently limited in its inspections of research reactors
  • Small amounts of material could be produced/diverted on a consistent basis
  • Require measurement of reactor characteristics

• Research reactors are abundant
  • 247 research reactors
  • May not be production facility
  • Indicative of state’s ambitions
NDA Techniques

Monitoring

Nuclear Properties

Photon Methods

Passive Techniques
- Total Photon Counting
- Spectroscopy
- Imaging

Calorimetry
- Attenuation

Active Techniques
- Total Neutron Counting
- Coincidence/\text{n Multiplicity}
- Photo-fission

Neutron Methods

Passive Techniques
- Die-Away
- Coincidence/\text{n Multiplicity}

Active Techniques
- Spectroscopy
- Resonance Absorption (LSDS)
- Imaging
- Delayed Fission Products

CRANK System
Equipment Requirements

- Shielding to surround detector system’s non-robust components
  - Previous INL experience will be vital

- Data acquisition system
  - National Instruments hardware or equivalent

![Graph showing % Deviation from β/I Ratio and Measurement Uncertainty (%) vs Sample Rate (Hz)]
Fuchs-Nordheim Derivation

\[
\frac{dP}{dt} = \frac{\rho(t) - \beta}{l} P(t)
\]

\[
\rho(t) = \rho_0 - \alpha T(t)
\]

\[
\frac{dT}{dt} = KP(t)
\]

\[
\frac{d\rho}{dt} = -\alpha \frac{dT}{dt} = -\alpha KP(t)
\]

\[
\frac{dP}{d\rho} = -\frac{1}{\alpha K} \cdot \frac{\rho(t) - \beta}{l}
\]

\[
\int_{P_0}^{P(t)} dP = -\frac{1}{\alpha K} \cdot \int_{\rho_0}^{\rho(t)} \frac{\rho'(t) - \beta}{l} d\rho'
\]
Fuchs-Nordheim Derivation

\[ P(t) - P_0 = \frac{-1}{2\alpha lK} \left( (\rho(t) - \beta)^2 - (\rho_0 - \beta)^2 \right) \]

\[ \frac{dP}{dt} = \frac{-(\rho(t) - \beta)}{\alpha lK} \]

\[ \frac{dP}{dt} = 0 \rightarrow \rho(t) = \beta \]

\[ P_{\text{max}} = \frac{(\rho_0 - \beta)^2}{2\alpha lK} \]
Fuchs-Nordheim Derivation

\[ P(t) = \frac{-1}{\alpha K} \frac{d\rho}{dt} \]

\[ \frac{d\rho}{dt} = \frac{1}{2l} \left( (\rho(t) - \beta)^2 - (\rho_0 - \beta)^2 \right) \]

\[ \frac{d\rho}{\left( (\rho(t) - \beta)^2 - (\rho_0 - \beta)^2 \right)} = \frac{1}{2l} dt \]

\[ \int_{\rho(t)}^{\rho'} \frac{d\rho'}{\left( (\rho'(t) - \beta)^2 - (\rho_0 - \beta)^2 \right)} = \int_{0}^{t} \frac{1}{2l} dt \]

\[ \int \frac{du}{u^2 - a^2} = -\frac{1}{a} \text{arc tanh} \left( \frac{u}{a} \right) + C \]

\[ \frac{1}{(\rho_0 - \beta)} \text{arc tanh} \left( \frac{\left( \rho(t) - \beta \right)}{(\rho_0 - \beta)} \right) = -\frac{t}{2l} \]
Fuchs-Nordheim Derivation

\[ \rho(t) = \beta - (\rho_0 - \beta) \tanh \left( \frac{(\rho_0 - \beta)t}{2l} \right) \]

\[ P(t) = \frac{-1}{\alpha K} \frac{d\rho}{dt} \]

\[ P(t) = \frac{(\rho_0 - \beta)^2}{2\alpha lK} \sech^2 \left( \frac{(\rho_0 - \beta)t}{2l} \right) \]

\[ P(t) = P_{\text{max}} \sech^2 \left( \frac{\rho_0 - \beta}{2l} t \right) \]
12 open-pool pulse research reactors in 12 NNWS

Austria  Germany  Slovenia
Bangladesh  Mexico  Thailand
Brazil  Nigeria  Turkey
Finland  Romania  Ukraine
Reactor Experiments

• Data from $0.50$ square waves in OSTR was collected previously
  o Reactor power was measured by fission chamber
  o Two reactor transients – Monday and Wednesday
    ▪ Monday, August 28, 2014 – Xenon buildup allowed to decay during weekend
    ▪ Wednesday, November 5th, 2014 – Core reactivity could change due to Xenon buildup

• Compared with PJ approximation for U-235
  o OSTR core has little burnup
Reactor Experiments

Graph showing the Power (Normalized) over Time (seconds) with different lines for $0.50 Monday, $0.50 Wednesday, and PJ Approx U-235 Only.
Closer to core
IAEA & Reactors

• Inspectors at reactor facilities are primarily focused on plutonium
  o The IAEA maintains records of the amount of plutonium produced at each reactor facility
    ▪ Specifically, Pu-239
  o Inspect fuel assemblies in spent fuel pools
  o Ensure documentation containing information about fuel rods is accurate (extra fuel pins, missing fuel pins, etc.).
  o Pu-239 is produced as part of normal reactor operations when U-238 absorbs a neutron and undergoes radioactive decay.